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Derivatives.

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In recent years there has been a considerable amount of research devoted to questions involving the derivative of a function of one real variable and its generalizations. This activity is due, in part, to the fundamental role played by the derivative in mathematics, and, also to the difficulty of some of the interesting unsolved problems related to derivatives. It seems appropriate that some of the results of this activity, along with some of the interesting but not-so-well-known earlier results, be brought together and examined in one place. This is one of the purposes of the present article.

In deciding which topics to include in this article, the authors give preference to ones which can be discussed without first having to develop a great deal of machinery. In addition, they have leaned towards topics in which recent work has been done and for which unsolved problems can be stated. The following topics are discussed: Continuity of the derivative; The structure of the sets $\{x; f'(x) > \alpha\}$ and $\{x; f'(x) < \alpha\}$; Derivatives almost everywhere and universal generalized antiderivatives; Dini derivatives; Approximate derivatives; Other generalizations of the derivative; Points of differentiability; Inversion of derivatives; Stationary sets and determining sets; Intervals of constancy; Monotonicity; Derivatives and Darboux functions of Baire class 1. A list of 216 references is added.

The article contains many interesting remarks and some new results. All the topics are presented very systematically and with a remarkable elegance. This article will undoubtedly stimulate new researches in this field.

Reviewed by *S. Marcus*