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Differentiation of integrals.

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The author gives a comprehensive survey of the existing results in the theory of differentiation of integrals, with a few omissions that are related to the work of A. Denjoy [*Amer. J. Math.* **73** (1951), 314–356; [MR0040385 \(12,685b\)](#)] and W. J. Trjitzinsky [*Théorie métrique dans les espaces où il y a une mesure*, Gauthier-Villars, Paris 1960; [MR0133419 \(24 3253\)](#)]. This is evidently due to the highly technical nature of the approach in these works. Also, no mention has been made of differentiation of integrals with vector-valued integrands.

Central to the theory of differentiation are the classical Vitali covering property and the Lebesgue density theorem and/or one of their many variants and abstractions—Morse halo property, strong Busemann-Feller property and Denjoy-Trjitzinsky forms of the Vitali property, etc.

The article consists of eight chapters and an extensive bibliography. The first three chapters are devoted to covering and density properties and differentiation of integrals in Euclidean spaces. Chapters IV and VII give some applications of these topics to multiple Fourier series, harmonic functions, vector analysis, etc. In Chapter V certain generalizations of classical results on differentiation of functions of one variable are presented. Chapter VI gives a summary of results on differentiation of integrals in abstract measure spaces. It includes sufficient conditions for the existence of a differentiation base, i.e., a family of sets endowed with a contraction property. Chapter VIII, the last chapter, is a list of problems that, according to the author, seem to be unsolved.

This is an excellent survey of the topic and will no doubt prove to be a valuable reference paper to both specialists and non-specialists in the field.

Reviewed by *M. Anvari*

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