Jean Dieudonné (1906-1992)
mathematician

Pierre CARTIER

Institut des Hautes Études Scientifiques
35, route de Chartres
91440 – Bures-sur-Yvette (France)

Août 2005

IHES/M/05/28

Jean Dieudonné (1906-1992) mathematician

Pierre Cartier

Jean Dieudonné has been one of the most influential French mathematicians during the 20^{th} century, especially through his association – even identification – with the Bourbaki group. An excellent biography has been written by his friend P. Dugac, a historian of science [4]. We shall retrace here his long and distinguished career.

The man and his life

He was born in 1906 in a priviledged family: his father, a self-made man, succeeded, after a modest beginning, at becoming the manager of a big complex in the textile industry, his mother has been an elementary school teacher. Despite the hardships of World War One – his native town of Lille was under the German ruling for four years – his family managed to grant him the best education, including a year in a British boarding school at the age of 12. He was admitted at 18 to the École Normale Supérieure, one of the most prestigious institutions of higher learning in France. There he started lifelong friendships with the best minds, especially the mathematicians Henri Cartan and André Weil. He obtained his Ph.D. at the age of 25, and then climbed smoothly and quickly the ladders of an academic career. After one year in Bordeaux, and four years in Rennes, he moved to Nancy where he was to stay from 1937 to 1952. Before reaching 35, he was promoted a full professor. He spent then seven years in the United States (University of Michigan, then Northwestern University). When he returned to France in

1959, he participated in the creation of the *Institut des Hautes Études Scientifiques* (I.H.É.S., Bures-sur-Yvette), where his collaboration with Alexander Grothendieck brought an instant fame to this new research institution. Never tired, he left the I.H.É.S. in 1964 to be the first Dean of Science at the newly created University of Nice, where the Mathematics Institute is named after him. The climax of his academic career was the organization of the World Congress of Mathematics (ICM70) in Nice in 1970. After this big success, he retired and devoted the rest of his life writing books.

The man was heavy and tall, with indomitable energy and optimism, but was not fond of physical exercise, except occasional hiking. His philosophy of life was that of stoicism: strong sense of public good, great personal courage, respect of laws and traditions until they conflict with moral values. He was hardworking, sleeping 5 to 6 hours every night, he was well organized, but was not an ascetic. He enjoyed good food, great wines, conversation with friends, and playing (well) the piano. In his last days, bedridden, he was happy to listen to one of the many records given to him as a gift for his 85$^{\text{th}}$ birthday, reading and commenting the musical score at the same time. To sum up, an attractive personality, faithful to his friends if somewhat inattentive to his family.

He was a social conservative, strongly believing in a social hierarchy based on meritocraty, and had many arguments with his friends most of them with a bent towards the left. He was not engaged in political activities, unlike his friends H. Cartan and L. Schwartz. But his strong sense of justice brought him twice into embarrassing and potentially dangerous situations, once to help the Czech opponents to the Soviet regime in 1979 and the second time on behalf of the communist mathematician L. Massera in Urugay a little later. He had strong opinions and expressed them loudly, but was intellectually very honest and accessible to a rational argument. He was not pursuing honors for their own sake, but did not fight shy of responsabilities. Hence, at the end of his life he was happy to become a member of the *Académie des Sciences* (Paris) and played an active role there, despite the initial youthful reservations of him and his friends from Bourbaki towards the *Académie* (after he had been elected in 1968, he managed to get all of them quickly into the *Académie*). He took very seriously his charge as a Dean of Science in Nice, from 1965 to 1968, at the height of the wave of student unrest.

He had a consuming love for science, and devoted his whole life to the pursuit of it. He told me how, in the gruesome conditions of war in 1940, he would find comfort and solace in studying mathematics and playing music.

I experienced the same situation years later, as did my Vietnamese friends.

The mathematician

Dieudonné wrote about 300 papers, most of them rather short, and published 26 books under his own name. A large part of the volumes of the Bourbaki treatise [2] – as well as numerous preliminary drafts – flew from his pen, and he published a series of 8 volumes with A. Grothendieck [3]. It is no exaggeration to estimate that he wrote 80 000 pages, at a rythm of 5 pages a day. Few authors are as prolific as Dieudonné!

He published in [27] a selection of his mathematical papers, containing an interesting notice [1] about his work. He contributed to many areas of mathematics, and he was rightly considered as *the* encyclopedic mathematician. His thesis and what came around is about the classical subject of zeroes of complex polynomials; he was able to prove a particular case of the famous *Bieberbach conjecture*: if a function $f(z)$ is given by a power series $\sum_{n=0}^{\infty} a_n z^n$ converging for $|z| < 1$, and furthermore if $f(z) \neq f(z')$ whenever $|z| < 1$, $|z'| < 1$ and $z \neq z'$, prove the inequality $|a_n| \leq n\,|a_0|$. He didn't pursue very long this line of research since the creation of Bourbaki diverted his interests towards topology and functional analysis. In the field of topology, he is credited with the definition of a *partition of unity*, a powerful tool to globalize properties of functions.

The mathematical contributions of Dieudonné can be summarized under four headlines.

A) *Functional analysis.* The theory of normed spaces was developed around 1920 by Banach and his school. Later on, around 1935, Toeplitz and Köthe studied in depth spaces consisting of sequences of numbers. Using the general notion of locally convex space introduced by von Neumann, Dieudonné extended in 1942 to this broader framework the results of Banach, mainly the duality, the weak convergence and the perturbation of operators. His major contribution came a little later (1949) as a long paper coauthored with L. Schwartz about the duality in the so-called (F)- and (LF)-spaces.

The purpose of this last paper was to provide theoretical foundations for the newly developed theory of distributions by L. Schwartz. But came an unexpected bonus. In 1949, A. Grothendieck, then 21 years old, came to Nancy to prepare his Ph.D. Dieudonné proposed to Grothendieck to answer the questions at the end of his paper with Schwartz. So far, the theory of topological vector spaces had been pursued single-handedly by Dieudonné, and found its way into the corresponding Book V of Bourbaki's treatise. But Grothendieck revolutionized the whole field, through a subtle analysis of the relations between compactness and duality, and through the definition of a new class of spaces, the *nuclear spaces.* In a sense, he killed the theory by solving all its problems, and his nuclear spaces proved to be a fantastic tool in the hands of Gelfand and his school (especially in probability and mathematical physics).

In recent years, there has been less emphasis on the general theory of locally convex spaces. Banach and Hilbert spaces remain at the core of Analysis, as well as scales of Hilbert spaces like the Sobolev spaces. Nevertheless, the ideas of duality, weak convergence and compactness remain central.

B) *Ring theory.* Dieudonné was the rare combination of an analyst and an algebraist. There are two main periods in his work on algebra: before and after 1950. Before 1950, he followed the new trend in algebra, started by the German school around E. Artin, E. Nœther and B.L. van der Waerden. The main emphasis was about structural properties of groups, rings, algebras, proved by axiomatic, intrinsic methods. This was exactly the spirit of the young N. Bourbaki (born in 1934). Among many results of Dieudonné which found their way into text-books, the most conspicuous ones are the definition of the *socle* of a ring, and of the *noncommutative determinant*, nowadays known as Dieudonné's determinant, with applications to differential operators. Following N. Jacobson's lead, Dieudonné ventured into the theory of (infinite) simple noncommutative rings, using the tool of weak continuity, and into a corresponding Galois theory; it has to be admitted that this was a blind alley.

C) *Classical groups.* That is his main contribution to pure algebra. Groups were invented in 1830 by Galois, who gave the first examples of matrix groups. Around 1860, C. Jordan developed extensively a theory of matrix groups over a finite field (Galois field) and proved the basic results about their generation, their simplicity and their automorphisms. Around 1900, E. Dickson extended these results by considering matrix groups with elements taken from a more or less arbitrary field. The methods of Jordan and

Dickson were highly computational, and full of special cases, and left aside some exceptional cases (fields of characteristic 2, for instance). Dieudonné took it to himself to streamline the proofs by using the new geometrical language afforded by vector spaces. In 1948, he summarized his first results in a wonderfully written booklet [6]. A little later, he completely settled the question of automorphisms in a short monograph [7]. For about 7 years, he continued to work steadily on this subject, discovering the great importance of the notions of *involution* and *transvection*. He emphasized the difference between isotropic and anisotropic quadratic forms. The notion of involution has been widely used in the theory of finite simple groups (Feit-Thomson theorem), and transvections are a forerunner of the unipotent algebraic groups.

In 1955, Dieudonné summarized his results in a book [8] which remains up to now the standard reference. The results of Dieudonné had been obtained by "elementary" methods, most notably a clever use of linear and multilinear algebra. Immediately after 1955, began a new development in the study of classical groups, using Lie theory and the new concept of algebraic groups (A. Borel, C. Chevalley). In his foreword to [8], Dieudonné more or less anticipated these new developments: the theory of groups is no more elementary but draws extensively on the tools of modern algebraic geometry (sheaves, schemes, etale cohomology,...).

D) *Formal groups.* This topic is by far the deepest and most imaginative creation of Dieudonné, realized when Dieudonné was nearing 50, supposedly the term for an active mathematical life. It can be seen as the creation of a differential calculus for groups over a field of characteristic $p > 0$ (possibly finite). The methods of calculus do not work, and one has to resort to pure algebra. There were a number of forerunners: a version of Taylor's formula in characteristic $p > 0$ due to Dieudonné himself, the ideas of Delsarte about convolution operators (as explained in Book IV, chapter 6 of Bourbaki's *Éléments*), a definition of the Lie algebra of a Lie group and its enveloping algebra in terms of distributions on the group (by L. Schwartz). But the impetus came from the book by Chevalley, in 1951, about algebraic groups. Chevalley had developed a purely algebraic version of Lie theory, but restricted to fields of characteristic 0. The case of characteristic $p > 0$ was "terra incognita".

In a long series of papers, published between 1954 and 1958, later on collected into a book [18], Dieudonné explored in depth this new world. The general theory was recasted by P. Cartier using a duality akin to that of

Schwartz, and this led to a broad extension of the notion of *Hopf algebra*, coming initially from problems in Topology. But the deepest part was a classification of the corresponding commutative groups, and the invention of the so-called *Dieudonné modules*. These modules proved to be a fundamental tool in algebraic geometry, in the hands of P. Cartier, P. Gabriel, Yu. Manin, J.-M. Fontaine, to name a few, and remain the object of extensive research.

Mathematical collaborations

More than through his research, the lasting influence of Dieudonné will be through his two major collaborations: with Bourbaki, and with Grothendieck.

I will not detail the often told story of Bourbaki. The group began to work in 1934, with J. Dieudonné, H. Cartan, J. Delsarte, C. Chevalley and A. Weil who remained associated to the project, and R. de Possel, Sz. Mandelbrojt, C. Ehresmann (also the geophysicist J. Coulomb) who left after a few years. The first book appeared in 1939, a "Fascicule de Résultats" for Set Theory, a "worker's manual". The publication was slowed down by the Second World War, and centered at first around General Topology and Algebra. Most of the notes published between 1935 and 1945 by Dieudonné answer questions which aroused during the discussions in the group. If the scientific leader of Bourbaki, "primus inter pares", was undoubtedly A. Weil, the engineer (and secretary) was Dieudonné. He was called the "adjudant" because he was the one to put some order in the rather hectic discussions, he would write the minutes of meetings, devise plans of action, write many preliminary drafts and the printed version, do the proof reading and the negociations with the publisher, supply the many exercises at the end of each chapter, make proposals for revisions and new printings, write most of the historical notes accompanying the formal exposition. Without him, Bourbaki would not exist, and he dominated the group from its inception up to 1960.

After he stepped down, a younger generation around J.-P. Serre, A. Borel, A. Grothendieck (for a while) and many other energetic collaborators, took over. For twelve years, this was the golden age of Bourbaki, with one or two influential books published every year. Then Bourbaki was trapped in a long legal battle with his publisher, ending in an "a la Pyrrhus victory". After a swan-song, around 1980, with seven (new or revised) books published in a few years, Bourbaki having exhausted its momentum and cut from its roots began a long decay, like an old rotten oak falling apart. There was no second Dieudonné to give a new energy to the project, and the times were different!

The second successful mathematical collaboration of Dieudonné was his association with A. Grothendieck. As told above, Grothendieck went to Nancy in 1949, and within 4 years completed his great work centered around topological vector spaces. From that time on, Dieudonné had great esteem and admiration for Grothendieck, despite flagrant differences in personality. Being stateless, Grothendieck could not get a permanent position in France, and spent a few years in the Americas (Brazil, then United States). He returned to France when the I.H.É.S. began its operation. There was no legal restriction there, and Dieudonné, who had been appointed the first professor in mathematics, insisted on having Grothendieck with him. Grothendieck had just made a drastic, unexpected but not illogical, change in his research, and was engaging himself in the reconstruction of algebraic geometry around the notion of *scheme*. This subject was quite far from the field of expertise of Dieudonné, but his flexible mind adapted immediately. As a result, a continuous flow of ideas from the brain of Grothendieck was transformed by Dieudonné into a painstakingly written version, to be printed in eight of the first issues of the newly founded “Publications Mathématiques de l'I.H.É.S.”. These books [3] are still today, forty years later, the foundation for this new subject. Unfortunately, in the early 1970's, Grothendieck became more and more unhappy with the world of mathematics, and his personal relationship with Dieudonné deteriorated.

The historian of mathematics

After 1970, Dieudonné devoted the greatest part of his activity to the history of mathematics. He had great assets for this entreprise: a broad knowledge of mathematics past and present, a flexible mind and a good sense of organization, an extreme rigour and attention to details. He had already some training, having written with A. Weil the historical vignettes for Bourbaki. He shared with Weil a certain view of history of mathematics which can be summarized as follows:

– no attention should be paid to biographical details, historical anecdotes, controversies about priorities;

– explain the development of the main ideas, how new concepts emerge, how problems are solved;

– stress the unity of mathematics, and its autonomy with respect to other sciences and technology.

All these statements are highly controversial, and represent the belief of a "pure" mathematician. But it has to be said that the books written by Dieudonné and Weil according to these principles are immensely successful in spite of their biases. The main reservation I would express is their implicit *teleology:* the development of mathematics is explained by the emergence of the concepts which led *by necessity* to the present point of view.

Let us describe his two main successes. There is first the "Abrégé d'Histoire des Mathématiques 1700-1900" (in two volumes) published in 1978 [24]. This is a collective work, sustained by the previous philosophy and strongly coordinated by Dieudonné. The audience should be teachers in high schools, but the level of abstraction and the sophistication of the mathematical argument are very demanding (especially the chapter on abelian integrals). The book gives a precise description of what has been achieved before the "modern" period in the first volume and hints at more recent results in the second one. The professional mathematician or historian will find a wealth of precisely documented history and exposition. A second, shortened, edition was aimed at a slightly larger audience.

The second success is the "History of Algebraic and Differential Topology" [30] published in 1989, a few years before his death. This book not only records the historical order of development, it is also one of the best available introduction to a very important part of contemporary mathematics, with a broad range of applications. This is the standard reference I recommend to my Ph.D. students. The book has been highly praised as the last symphony of a great master, something like the 9^{th} Symphony of Beethoven.

Drawing on his own fields of research, he wrote also an excellent "History of Functional Analysis" [26] in 1981, with great emphasis on topological vector spaces and distributions (as expected). Finally, there is a "History of Algebraic Geometry" [28] published in 1985, where starting from Poncelet and Riemann, he moves through the Italian school (Enriques, Severi,...) to the fusion of algebra and algebraic geometry by the German school of the 1930's to reach finally the modern synthesis of Grothendieck. The last chapter is a beautiful and well-informed panorama of present algebraic geometry, with its many open questions.

Notice that all these books were written when Dieudonné was over 70!

Expository work and polemics

From his youth on, Dieudonné was fascinated by dictionaries and encyclopediae, and liked to draw long lists of topics or names. There is ample testimony of this bent in his bureaucratic reports, whether for the University of Nice when he was the Dean of Science, or for the Academy of Sciences. This tendency developed when he grew up, and prompted him to numerous projects.

He wrote three text-books for mathematicians. There is first "Algèbre Linéaire et Géométrie Élémentaire" [9] published in 1964, as part of the ongoing fight for the reform of mathematics curriculum in high school ("Modern Math. debate"). Dieudonné argues that a student should learn first the notions of vector space and quadratic form, and then take this as a basis for geometry. There seems to be no doubt that vector algebra, including the scalar product (that is Thales' and Pythagora's theorems revisited) is the basis of euclidean geometry. The problem is to make this palatable for kids at the age of 14 to 16!

His "Calcul Infinitesimal" [14] published in 1968 addresses the needs of a good undergraduate – and his instructor – in Calculus. This book is well-written, well-arranged and classroom-tested. It compares to the best classics like Whittaker and Watson.

The third project is much more ambitious. In the nine volumes of his "Éléments d'Analyse", he gives a complete coverage, in depth, of advanced calculus. He describes the basic results about Banach and Hilbert spaces, and spectral theory, as well as differentiable manifolds (riemannian or not), distributions and differential operators, Lie groups, pseudo-differential operators and differential topology. The complete tool-box of the practising analyst and/or differential geometer! All this with relentless precision and rich details of information, as well as numerous exercises. One can imagine that these books represent the original project of "Traité d'Analyse" of Bourbaki as envisioned by one of the founding fathers.

I should add two surprising little books. In "Panorama des Mathématiques Pures" [21], he gives a strange classification of mathematical subjects, weighed according to the number of lectures at the Bourbaki Seminar devoted to them.

As a result, the basic subjects treated in depth in the Bourbaki "Éléments" have zero density, as well as such big fields like Numerical Analysis or Combinatorics. Applied mathematics doesn't exist! This is only to show the strong biases of Dieudonné and his followers. Another book "Pour l'honneur de l'esprit humain" [29] aims at conveying to a general audience the meaning of doing mathematics. The author's enthousiasm made him overnight a star in the best talk-show of French TV!

To be complete, we should mention his contributions to various encyclopedic projects. For instance, he was the chief-editor of the mathematics section of *Encyclopedia Universalis* whose ambition was to compete with *Encyclopedia Britannica* (the two merged recently!). He wrote also numerous entries for the *Dictionary of Scientific Biography.*

It will be no surprise to the reader that like his predecessor d'Alembert, Dieudonné liked to engage in *controversies*, whether about the teaching of mathematics, the purity of mathematics, the organization of Universities, the logical foundations of mathematics, the meaning of history and philosophy of science. He wrote at length on all these subjects, and gave numerous lectures. Before 1960, he was the acknowledged spokesman for Bourbaki, and defended with great strength and talent the views of Bourbaki about the meaning of mathematics and the best way to practise it and write it. For a while, "À bas Euclide" was his motto. Let us conclude with "Vive Dieudonné!".

References

[1] J. Dieudonné, *Notices sur les travaux scientifiques*, reproduite dans [27], vol. II, pp. 693-722.

[2] N. Bourbaki, *Éléments de Mathématique*, 27 vol., Hermann et Masson, Paris, 1939-1998.

[3] A. Grothendieck, *Éléments de Géométrie Algébrique* (rédigés avec la collaboration de J. Dieudonné), Chap. 0-I-II-III-IV, Publ. Math. I.H.É.S., volumes 4, 8, 11, 17, 20, 24, 28, 32 (1460 pages), P.U.F., 1960-1967.

[4] P. Dugac, *Jean Dieudonné, mathématicien complet*, Editions Jacques Gabay, Paris, 1995.

Books by J. Dieudonné

[5] Théorie analytique des polynômes d'une variable (Mémorial des Sciences Mathématiques, Fasc. XCIII), Gauthier-Villars, Paris, 1939.

[6] Sur les groupes classiques (Collection Actualités scientifiques et industrielles n° 1040), Hermann, Paris, 1948.

[7] On the automorphisms of the classical groups, Mem. A.M.S., vol. 2, 1951.

[8] La géométrie des groupes classiques (Collection Ergebnisse der Mathematik, Neue Folge, Heft 5), Springer, Heidelberg, 3e éd., 1971.

[9] Algèbre linéaire et géométrie élémentaire (Collection Enseignement des Sciences), Hermann, Paris, 1964.

[10] Fondements de la géométrie algébrique moderne, Les Presses de l'Université de Montréal, Montréal, 1964; 2e édition, 1966.

[11] Topics in Local Algebra, Notre Dame Math. Lectures n° 10, Notre Dame Univ. Press, 1967.

[12] Éléments d'analyse. Tome I, Fondements de l'analyse moderne. Chapitres I à XI (Collection Cahiers Scientifiques, Fasc. XXVIII), Gauthier-Villars, Paris, nouvelle édition revue et corrigée, nouveau tirage 1971.

[13] Éléments d'analyse. Tome II. Chapitres XII à XV (Collection Cahiers Scientifiques, Fasc. XXXI), Gauthier-Villars, Paris, 2e édition, 1974.

[14] Calcul infinitésimal (Collection Méthodes), Hermann, Paris, 1968.

[15] Éléments d'analyse. Tome III. Chapitres XVI et XVII (Collection Cahiers Scientifiques, Fasc. XXXIII), Gauthier-Villars, Paris, 2e édition, 1974.

[16] Éléments d'analyse. Tome IV. Chapitres XVIII à XX (Collection Cahiers Scientifiques, Fasc. XXXIV), Gauthier-Villars, Paris, 1971.

[17] Invariant theory, old and new (with J. Carrell), Academic Press, New York, 1971.

[18] Introduction to the theory of formal Lie groups, Marcel Dekker, New York, 1973.

[19] Éléments d'analyse. Tome V. Chapitre XXI (Collection Cahiers Scientifiques, Fasc. XXXVIII), Gauthier-Villars, Paris, 1975.

[20] Éléments d'analyse. Tome VI. Chapitre XXII (Collection Cahiers Scientifiques, Fasc. XXXIX), Gauthier-Villars, Paris, 1975.

[21] Panorama des mathématiques pures : le choix bourbachique (Collection "Discours de la méthode"), Gauthier-Villars, Paris, 1977.

[22] Éléments d'analyse. Tome VII. Chapitre XXIII (Collection Cahiers Scientifiques, Fasc. XL), Gauthier-Villars, Paris, 1978.

[23] Éléments d'analyse. Tome VIII. Chapitre XXIII, deuxième partie (Collection Cahiers Scientifiques, Fasc. XLI), Gauthier-Villars, Paris, 1978.

[24] Abrégé d'Histoire des Mathématiques 1700-1900 (avec la collaboration de P. Dugac, W.J. et F. Ellison, J. Guérindon, M. Guillaume, G. Hirsch, C. Houzel, P. Libermann, M. Loève, J.-L. Verley), 2 vol., Hermann, Paris, 1978 (traduit en allemand et en japonais).

[25] Éléments d'analyse. Tome IX. Chapitre XXIV (Collection Cahiers Scientifiques, Fasc. XLII), Gauthier-Villars, Paris, 1981.

[26] History of Functional Analysis (North Holland Math. Studies, n° 49), North Holland, Amsterdam-New York-Oxford, 1981.

[27] Choix d'Œuvres Mathématiques, 2 vol., Hermann, Paris, 1981.

[28] Cours de géométrie algébrique, tome I, P.U.F., Paris, 1974; trad. anglaise: History of Algebraic Geometry, Wadsworth, Monterey, 1985.

[29] Pour l'honneur de l'esprit humain, Hachette, Paris, 1987 (traduit en anglais, espagnol, italien, japonais et portugais).

[30] A History of Algebraic and Differential Topology 1900-1960, Birkhäuser, Boston, 1989.

Curriculum vitae of Jean Dieudonné

Born in Lille (France), July 1, 1906.

Fellow of Bembridge School (UK) 1919/20.

Lycée Faidherbe at Lille, 1920-23.

First Prize in Mathematics (Concours Général), 1923.

Baccalauréat, 1923.

Student École Normale Supérieure, 1924-27.

Agrégé des Sciences Mathématiques, 1927.

Fellowship at Princeton University (USA) 1928/29.

Teaching assistant (École Normale Supérieure), 1929/30.

Rockefeller foundation fellowship; Berlin and Zürich, 1930/31.

Docteur ès Sciences Mathématiques (Ph.D.), 1931.

"Chargé de cours" at Faculté des Sciences Bordeaux, 1932/33.

"Chargé de cours", promoted "Maître de Conférences" at Faculté des Sciences Rennes, 1933-37.

"Maître de Conférences", promoted professor at Faculté des Sciences Nancy, 1937-52.

Professor at Michigan University (USA), 1952/53.

Professor at Northwestern University (USA), 1953-59.

Professor at I.H.É.S., Bures-sur-Yvette (France), 1959-64.

Professor at Nice University, 1964-69.

Dean of Science at Nice University, 1964-68.

Honorary Professor at Nice University, 1970.

Deceased, November 29, 1992.

Honors

Officier de la Légion d'Honneur.

Commandeur des Palmes Académiques.

Corresponding member Académie des Sciences, 1965.

Gaston Julia Prize, 1966.

Member Académie des Sciences, 1968.

Foreign member Academy of Sciences (Madrid, Spain).

Foreign member Academy of Sciences, Belgium.

Visiting professorships

University of Sao-Paulo (Brazil), 1946-48.

Columbia University (USA), Summer 1950.

Johns Hopkins University (USA), Spring 1951.

University of Rio-de-Janeiro (Brazil), Summer 1952.

University of Pisa (Italy), Spring 1961.

University of Buenos-Aires (Argentina), Summer 1961.

University of Maryland (USA), Spring 1962.

Tata Institute at Bombay, Winter 1963.

Visiting program in Japan, May 1964.

Visiting program in the USA, Spring 1965.

Notre-Dame University (USA), Fall 1966.

Washington University (USA), Summer 1967.

Notre-Dame University (USA), 1969-71.