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Darboux transformations.

Trans. Amer. Math. Soc. **128** 1967 103–111

A notion of Darboux transformation which includes, as special cases, notions previously considered by Goffman, Waterman and Neugebauer [C. Goffman and D. Waterman, Proc. Amer. Math. Soc. **12** (1961), 116–121; MR0120327 (22 #11082); C. J. Neugebauer, Trans. Amer. Math. Soc. **107** (1963), 30–37; MR0148811 (26 #6315)], similar to one considered by L. Mišik [Mat.-Fyz. Časopis Sloven. Akad. Vied **14** (1964), 44–49; MR0181719 (31 #5946)], is considered for functions taking on values in a separable metric space X^*. Let X be a Euclidean space and $\mathcal{B}$ a topological base for X all of whose members are connected sets. A transformation f mapping X into X^* is called Darboux $(\mathcal{B})$ provided $f(\overline{U})$ is connected for all $U \in \mathcal{B}$. The base $\mathcal{B}$ is said to satisfy $(*)$ provided any transformation of an element of $\mathcal{B}$ is in $\mathcal{B}$, $(**)$ provided for any $x \in X$ and $U \in \mathcal{B}$ with $x \in \overline{U}$, there is some $V \in \mathcal{B}$ such that $x \in \overline{V}$ and $\overline{V} - \{x\} \subseteq U$, and (**+) provided that $(**)$ holds and for each $\varepsilon > 0$, V may be taken in $(**)$ to have diameter less than ε. Under the assumption that $\mathcal{B}$ fulfills $(*)$ and $(**)$, a necessary and sufficient condition that a real-valued function be Darboux$(\mathcal{B})$ is established. With a properly selected use of $(*)$, $(**)$ and (**+), relations are established between various conditions concerning Baire type 1 transformations from X to X^*. The conditions considered are analogous to conditions considered by Z. Zahorski [Trans. Amer. Math. Soc. **69** (1950), 1–54; MR0037338 (12,247c)], W. H. Young [Rend. Circ. Mat. Palermo **24** (1907), 187–192], I. M. Maksimov [Prace Mat. Fiz. **43** (1936), 241–265] and H. W. Ellis [Canad. J. Math. **3** (1951), 471–485; MR0043872 (13,332d)] for ordinary Darboux functions. A necessary and sufficient condition that a Darboux $(\mathcal{B})$ transformation of X into X^* be continuous is established. Examples are given to illustrate the need for various hypotheses in the theorems.

Reviewed by *D. W. Solomon*

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Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.