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**Rethinking the elementary real analysis course.**
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This is an important paper for mathematical education and should be read by anyone planning a course or about to write a text on analysis or calculus. Ever since Henstock rather brashly said "Lebesgue is dead!" at the Stockholm meeting in 1962 there have been many hardy souls, colleagues, students and disciples of either Henstock or Kurzweil suggesting the more reasonable and practical "Riemann is dead!". Many papers have been written that make this point, as well as books by both Henstock and Kurzweil, the author of the present paper, Bartle, DePree and Schwarz, Leader, Lee and Výborný, Mawhin, McLeod, McShane, and no doubt others; as well Dieudonné, in the book that is quoted at the beginning of the present article, made the same point from a completely different point of view. All argue that by now the teaching of both the Riemann and Riemann-Stieltjes integrals should cease; these integrals should be replaced by the Newton and Cauchy integrals in elementary calculus courses and by the generalized Riemann integral in elementary analysis courses, the latter being taught in a way that would lead into Lebesgue theory in more advanced courses. To date the conservative nature of academia has not heard these arguments.

The present paper suggests a very attractive method for the elementary analysis course mentioned above. The author has a rather extreme aversion to the plethora of gauges and tags that are the norm in most approaches to the generalized Riemann integral. Instead he suggests that the basis of the analysis course should be Cousin's Lemma. This very elementary formulation of the completeness axiom of the real line allows for proofs of all the properties normally deduced from that axiom, including the properties of continuous functions normally considered in basic analysis courses—proofs that are both simple and transparent. It further leads very naturally into the generalized Riemann integral and later to measure theory if that is desired. The paper is very clearly written and is very persuasive but is not an easy read, especially towards the end, and the reader may need the help of a standard book on the generalized Riemann integral. In addition the author shows that a simple recent extension of Cousin's Lemma will allow some very neat proofs of more subtle properties that may or may not be appropriate for the first analysis course but certainly would allow for the progression to a more advanced course and to measure theory.

It is difficult to change a very well-established academic and publishing tradition but the reviewer hopes that this article will start at least the beginnings of a change. There are already non- or semi-commercial web texts that are moving in this direction, some under the influence of the Dump-the-Riemann-Integral-Project (see http://classicalrealanalysis.com/drip.aspx) and others less radical being produced by the Trillia Group (http://www.trillia.com). These should all be explored by anyone teaching in this field.

Reviewed by *P. S. Bullen*

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*Note: This list reflects references listed in the original paper as accurately as possible with no attempt to correct errors.*