PREFACE

Preface to the second edition

This second edition is a corrected, revised, and reprinted version of our original textbook. We are particularly grateful to readers who have sent in suggestions for corrections. Among them we owe a huge debt to R. B. Burckel (Kansas State University). Many of his corrections and suggestions are incorporated in this new edition. Thanks too to Keith Yates (Manchester Metropolitan University) who, while working on some of the more difficult problems, found some further errors.

Original Preface to first edition

In teaching first courses in real analysis over the years, we have found increasingly that the classes form rather heterogeneous groups. It is no longer true that most of the students are first-year graduate students in mathematics, presenting more or less common backgrounds for the course. Indeed, nowadays we find diverse backgrounds and diverse objectives among students in such classes. Some students are undergraduates, others are more advanced. Many students are in other departments, such as statistics or engineering. Some students are seeking terminal master's degrees; others wish to become research mathematicians, not necessarily in analysis.

We have tried to write a book that is suitable for students with minimal backgrounds, one that does not presuppose that most students will eventually specialize in analysis.

We have pursued two goals. First, we would like all students to have an opportunity to obtain an appreciation of the tools, methods, and history of the subject and a sense of how the various topics we cover develop naturally. Our second objective is to provide those who will study analysis further with the necessary background in measure, integration, differentiation, metric space theory, and functional analysis.

To meet our first goal, we do several things. We provide a certain amount of historical perspective that may enable a reader to see why a theory was needed and sometimes, why the researchers of the time had difficulty obtaining the "right" theory. We try to motivate topics before we develop them and try to motivate the proofs of some of the important theorems that students often find difficult. We usually avoid proofs that may appear "magical" to students in favor of more revealing proofs that may be a bit longer. We describe the interplay of various subjects—measure, variation, integration, and differentiation. Finally, we indicate applications of abstract theorems such as the contraction mapping principle, the Baire category theorem, Ascoli's theorem, Hahn-Banach theorem, and the open mapping theorem, to concrete settings of various sorts.

We consider the exercise sections an important part of the book. Some of the exercises do no more than ask the reader to complete a proof given in the text, or to prove an easy result that we merely state. Others involve simple applications of the theorems. A number are more ambitious. Some of these exercises extend the theory that we developed or present some related material. Others provide examples that we believe are interesting and revealing, but may not be well known. In general, the problems at the ends of the chapters are more substantial. A few of these problems can form the basis of projects for further study. We have marked exercises that are referenced in later parts of the book with a $\diamondsuit$ to indicate this fact.

When we poll our students at the beginning of the course, we find there are a number of topics that some students have seen before, but many others have not. Examples are the rudiments of metric space theory, Lebesgue measure in $\mathbb{R}^1$, Riemann–Stieltjes integration, bounded variation and the elements of set theory (Zorn's lemma, well-ordering, and others). In Chapter 1, we sketch some of this material. These sections can be picked up as needed, rather than covered at the beginning of the course. We do suggest that the reader browse through Chapter 1 at the beginning, however, as it provides some historical perspective.

Text Organization

Many graduate textbooks are finely crafted works as intricate as a fabric. If some thread is pulled too severely, the whole structure begins to unravel. We have hoped to avoid this. It is reasonably safe to skip over many sections (within obvious limitations) and construct a course that covers your own choice of topics, with little fear that the student will be forced to cross reference back through a maze of earlier skipped sections.

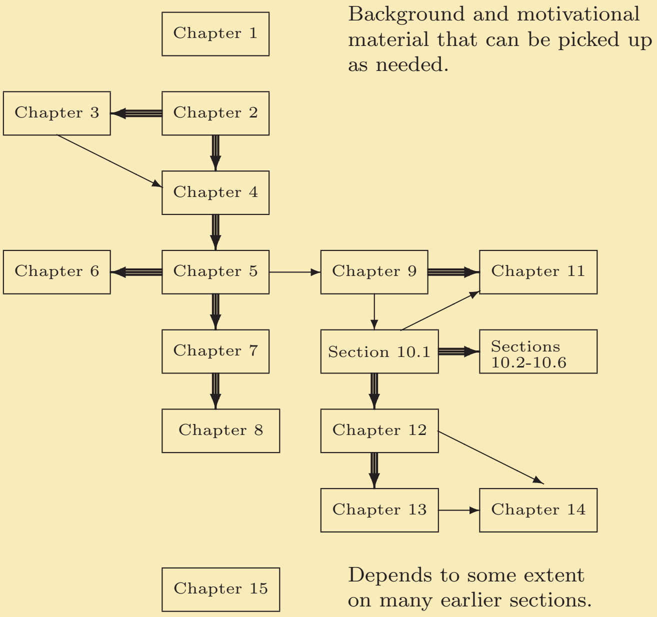
A word about the order of the chapters. The first chapter is intended as background reading. Some topics are included to help motivate ideas that reappear later in a more abstract setting. Zorn's lemma and the axiom of choice will be needed soon enough, and a classroom reference to Sections 1.3, 1.5 and 1.11 can be used.

The course can easily start with the measure theory of Chapter 2 and proceed from there. We chose to cover measure and integration before metric space theory because so many important metric spaces involve measurable or integrable functions. The rudiments of metric space theory are needed in Chapter 3, however, so we begin that chapter with a short section containing the necessary terminology.

Instructors who wish to emphasize functional analysis and reach Chapter 9 quickly can do so by omitting much of the material in the earlier chapters. One possibility is to cover Sections 2.1 to 2.6, 4.1, 4.2, and Chapter 5 and then proceed directly to Chapter 9. This will provide enough background in measure and integration to prepare the student for the later chapters.

Chapter 6 on the Fubini and Tonelli theorems is used only occasionally in the sequel (Sections 8.4 and 13.9). This is presented from the outer measure point of view because it fits better with the philosophy developed in Chapters 2 and 3. One can substitute any treatment in its place. Chapter 11 on analytic sets is not needed for the later chapters, and is presented as a subject of interest on its own merits. Chapter 13 on the L_p–spaces can be bypassed in favor of Chapter 14 or 15 except for a few points. Chapter 14 on Hilbert space could be undertaken without covering Chapters 12 and 13 since all material on the spaces ℓ_2 and L_2 is repeated as needed. Chapter 15 on Fourier series does not need the Hilbert space material in order to work, but, since it is intended as a showplace for many of the methods, it does draw on many other chapters for ideas and techniques.

The dependency chart gives a rough indication of how chapters depend on their predecessors. A strong dependency is indicated by a bold arrow, a weaker one by a fine arrow. The absence of an arrow indicates that no more than peripheral references to the earlier chapters are involved. Even when a strong dependency is indicated, the omission of certain sections near the en d of a chapter should not cause difficulties in later chapters. In addition, we have provided a number of concrete applications of abstract theorems. Many of these applications are not needed in later chapters. Thus an instructor who wishes to include material from all chapters in a year course for reasonably prepared students can do so by



1. Omitting some of the less central material such as 3.8 to 3.10, 5.10, 7.6 to 7.8, 8.4 to 8.7, 9.14 to 9.15, 10.2 to 10.6, and various material from the remaining chapters.

2. Sampling from the applications in Sections 9.8, 9.12, 9.14, 10.2 to 10.6, and 12.6.