# PREFACE

## Preface to the second edition (January 2008)

This edition differs from the original 2001 version only in that we corrected a number of misprints and other errors. We are grateful to the many users of that version for notifying us of errors they found. We would like to make special mention of Richard Delaware (University of Missouri-Kansas City), and Steve Agronsky (California State Polytechnic University, San Luis Obispo), both of whom went through the entire first edition, made many helpful suggestions, and found numerous errors.

### Original Preface (2001)

University mathematics departments have for many years offered courses with titles such as *Advanced Calculus* or *Introductory Real Analysis*. These courses are taken by a variety of students, serve a number of purposes, and are written at various levels of sophistication. The students range from ones who have just completed a course in elementary calculus to beginning graduate students in mathematics. The purposes are multifold:

1. To present familiar concepts from calculus at a more rigorous level.
2. To introduce concepts that are not studied in elementary calculus but that are needed in more advanced undergraduate courses. This would include such topics as point set theory, uniform continuity of functions, and uniform convergence of sequences of functions.
3. To provide students with a level of mathematical sophistication that will prepare them for graduate work in mathematical analysis, or for graduate work in several applied fields such as engineering or economics.
4. To develop many of the topics that the authors feel all students of mathematics should know.

There are now many texts that address some or all of these objectives. These books range from ones that do little more than address objective (1) to ones that try to address all four objectives. The books of the first extreme are generally aimed at one-term courses for students with minimal background. Books at the other extreme often contain substantially more material than can be covered in a one-year course.

The level of rigor varies considerably from one book to another, as does the style of presentation. Some books endeavor to give a very efficient streamlined development; others try to be more user friendly. We have opted for the user-friendly approach. We feel this approach makes the concepts more meaningful to the student.

Our experience with students at various levels has shown that most students have difficulties when topics that are entirely new to them first appear. For some students that might occur almost immediately when rigorous proofs are required, for example, ones needing $\varepsilon$-$\delta$ arguments. For others, the difficulties begin with elementary point set theory, compactness arguments, and the like.

To help students with the transition from elementary calculus to a more rigorous course, we have included motivation for concepts most students have not seen before and provided more details in proofs when we introduce new methods. In addition, we have tried to give students ample opportunity to see the new tools in action.

For example, students often feel uneasy when they first encounter the various compactness arguments (Heine-Borel theorem, Bolzano-Weierstrass theorem, Cousin's lemma, introduced in Section 4.5). To help the student see why such theorems are useful, we pose the problem of determining circumstances under which local boundedness of a function $f$ on a set $E$ implies global boundedness of $f$ on $E$. We show by example that some conditions on $E$ are needed, namely that $E$ be closed and bounded, and then show how each of several theorems could be used to show that closed and boundedness of the set $E$ suffices. Thus we introduce students to the theorems by showing how the theorems can be used in natural ways to solve a problem.

We have also included some optional material, marked as "Advanced" or "Enrichment" and flagged with the symbol ✂.

## Enrichment

We have indicated as "Enrichment"' some relatively elementary material that could be added to a longer course to provide enrichment and additional examples. For example, in Chapter 3 we have added to the study of series a section on infinite products. While such a topic plays an important role in the representation of analytic functions, it is presented here to allow the instructor to explore ideas that are closely related to the study of series and that help illustrate and review many of the fundamental ideas that have played a role in the study of series.

## Advanced

We have indicated as "Advanced" material of a more mathematically sophisticated nature that can be omitted without loss of continuity. These topics might be needed in more advanced courses in real analysis or in certain of the marked sections or exercises that appear later in this book. For example, in Chapter 2 we have added to the study of sequence limits a section on lim sups and lim infs. For an elementary first course this can be considered somewhat advanced and skipped. Later problems and text material that require these concepts are carefully indicated. Thus, even though the text carries on to relatively advanced undergraduate analysis, a first course can be presented by avoiding these advanced sections.

We apply these markings to some entire chapters as well as to some sections within chapters and even to certain exercises. We do not view these markings as absolute. They can simply be interpreted in the following ways. Any unmarked material will not depend, in any substantial way, on earlier marked sections. In addition, if a section has been flagged and will be used in a much later section of this book, we indicate where it will be required.

The material marked "Advanced" is in line with goals (2) and (3). We resist the temptation to address objective (4). There are simply too many additional topics that one might feel every student should know (e.g., functions of bounded variation, Riemann-Stieltjes and Lebesgue integrals). To cover these topics in the manner we cover other material would render the book more like a reference book than a text that could reasonably be covered in a year. Students who have completed this book will be in a good position to study such topics at rigorous levels.

We include, however, a chapter on metric spaces. We do this for two reasons: to offer a more general framework for viewing concepts treated in earlier chapters, and to illustrate how the abstract viewpoint can be applied to solving concrete problems. The metric space presentation in Chapter 13 can be considered more advanced as the reader would require a reasonable level of preparation. Even so, it is more readable and accessible than many other presentations of metric space theory, as we have prepared it with the assumption that the student has just the minimal background. For example, it is easier than the corresponding chapter in our graduate level text (*Real Analysis*, Prentice Hall, 1997) in which the student is expected to have studied the Lebesgue integral and to be at an appropriately sophisticated level.

## The Exercises

The exercises form an integral part of the book. Many of these exercises are routine in nature. Others are more demanding. A few provide examples that are not usually presented in books of this type but that students have found challenging, interesting, and instructive.

Some exercises have been flagged with the ✂ symbol to indicate that they require material from a flagged section. For example, a first course is likely to skip over the section on lim sups and lim infs of sequences. Exercises that require those concepts are flagged so that the instructor can decide whether they can be used or not. Generally, that symbol on an exercise warns that it might not be suitable for routine assignments.

The exercises at the end of some of the chapters can be considered more challenging. They include some Putnam problems and some problems from the journal *American Mathematical Monthly*. They do not require more knowledge than is in the text material but often need a bit more persistence and some clever ideas. Students should be made aware that solutions to Putnam problems can be found on various Web sites and that solutions to *Monthly* problems are published; even so, the fun in such problems is in the attempt rather than in seeing someone else's solution.

## Designing a Course

We have attempted to write this book in a manner sufficiently flexible to make it possible to use the book for courses of various lengths and a variety of levels of mathematical sophistication.

Much of the material in the book involves rigorous development of topics of a relatively elementary nature, topics that most students have studied at a nonrigorous level in a calculus course. A short course of moderate mathematical sophistication intended for students of minimal background can be based entirely on this material. Such a course might meet objective (1).

We have written this book in a leisurely style. This allows us to provide motivational discussions and historical perspective in a number of places. Even though the book is relatively large (in terms of number of pages), we can comfortably cover most of the main sections in a full-year course, including many of the interesting exercises.

Instructors teaching a short course have several options. They can base a course entirely on the unmarked material of Chapters 1, 2, 4, 5, and 7. As time permits, they can add the early parts of Chapters 3 and 8 or parts of Chapters 11 and 12 and some of the enrichment material.

## Background

We should make one more point about this book. We do assume that students are familiar with nonrigorous calculus. In particular, we assume familiarity with the elementary functions and their elementary properties. We also assume some familiarity with computing derivatives and integrals. This allows us to illustrate various concepts using examples familiar to the students. For example, we begin Chapter 2, on sequences, with a discussion of approximating $\sqrt{2}$ using Newton's method. This is merely a motivational discussion, so we are not bothered by the fact that we don't treat the derivative formally until Chapter 7 and haven't yet proved that $\frac{d}{dx}(x^2 - 2) = 2x$. For students with minimal background we provide an appendix that informally covers such topics as notation, elementary set theory, functions, and proofs.

## Acknowledgments

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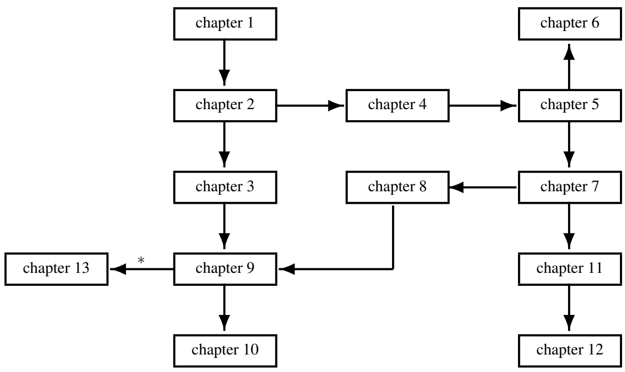


**Figure 0.1.** Chapter Dependencies (Unmarked Sections).